Method of Division of Motions for Control of Multi-Channel Linear Dynamic Objects

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Abstract: **This paper prolongs the research of method of division of motions. It is applied to the multi-channel case, when object has several inputs and outputs, which influence is interconnected. Such systems are treated as MIMO systems, which mean "many inputs and many outputs". This paper uses the Lemmas and Theorems proved in previous paper.**

Key words: **division motions method, localization method, feedback, control, automatics, dynamic error, static error, MIMO**

INTRODUCTION

This paper is development of the Localization approach [1] and Method of division of motion [2] for multi-channel case. Multi-channel case (MIMO) is very relevant for science, industry, machatronics and technology [3–7].

Multi-channel systems are assumed to be systems of automatic control of objects with r output values dependent on m input variables. In this case, the available cross relations do not allow considering the object as a set of single channel objects. If $r > m$, then $r - m$ output values remain uncontrollable. If $r < m$, then $m - n$ input parameters should be either fixed or dependent on the rest ones. Further we take $r = m$.

The purpose of control is to provide the property:

$$\lim_{t \to \infty} Y(t) = V(t)$$

where $Y(t)$ is an output vector of dimension m, $V(t)$ is a prescription vector of the same dimension.

Let us use the object description as matrix equation in the operator form:

$$B(s)Y(s) = A(s)U(s). \quad (1)$$

Here $U(s)$ is m-dimensional control vector, $A(s) = [a_{ij}(s)]$ and $B(s) = [b_{ij}(s)]$ are polynomial non-degenerate matrix of dimension $m \times m$, i. e. $\det A(s) \neq 0$, $\det B(s) \neq 0$.

It is assumed, that the greatest common divisor of elements in each said matrix is unity.

We require additionally that matrix $B(s)$ be diagonal one, i. e. $B(s) = diag\{b_i(s)\}$. It means

$$b_{ij}(s) = \begin{cases} b_i(s), & \forall i = j, \\ 0, & \forall i \neq j. \end{cases}$$

If it is not so, we multiply equation (1) by $B^{-1}(s)$ from the left side and with the introducing of the designation $W(s) = B^{-1}(s)A(s)$ we get

$$Y(s) = W(s)U(s). \quad (2)$$

Here $W(s) = [w_{ij}(s)]$ is matrix transfer function.

Let us find out the least common multiple (LCM) of denominator for scalar transfer function of each line:

$$b_i^*(s) = LCM\{w_{i1}(s), w_{i2}(s), ... w_{im}(s)\},$$

and if we multiply the equation (40) on the left by the matrix $B^*(s) = diag\{b_i^*(s)\}$, we get

$$B^*(s)Y(s) = B^*(s)W(s)U(s) = A^*(s)U(s),$$

Here $A^*(s) = B^*(s)W(s)$ is polynomial matrix. If the object description is given in terms of transfer functions (40), it is also reduced to the form (39).

The desired dynamic equation is

$$Q(s)Y(s) = V(s). \quad (2)$$

Here, $Q(s) = diag\{q_i(s)\}$ is diagonal matrix. Let us construct the regulator of the form

$$C(s)Y(s) + D(s)Z(s) = V(s), \quad (4)$$

where $D(s) = diag\{d_i(s)\}$, $C(s) = diag\{c_i(s)\}$ and $Z(s)$ is determined by the following equation:

$$U(s) = kA^{-1}(s)[Z(s) - R(s)Y(s)]. \quad (5)$$

Here, $R(s) = diag\{r_i(s)\}$ is diagonal matrix too, k is gain in the feedback loop (large coefficient).

1. MULTICHANNEL SYSTEMS: RAISING OF THE ORDER

Let us denote $m_i = \deg[d_i(s)]$ a power of polynomial $d_i(s)$.

Correspondingly, $n_i = \deg[r_i(s)]$, $n_i - 1 = \deg[c_i(s)]$. We note that the matrices $C(s)$ and $D(s)$ are not-degenerate, i. e.

$$c_i \neq 0, \; d_i \neq 0, \forall i = 1, ... m. \quad (6)$$

The value $n_i + m_i$ is the power of the characteristic polynomial of i-th circuit of the system "object + regulator" without taking into account fast motions. It is assumed that there are no

roots of equation $\det A(s) = 0$ in the right half of the complex plane. Otherwise the regulator will be unstable.

Lemma 1.

With the growth of k the system (1), (2), (4) is arbitrarily exactly described with the equation (3) where the matrix $Q(s)$ elements are determined as

$$q_i(s) = s^n d_i(s) + c_i(s) = \sum_{j=0}^{n_i+m_i} q_{ij} s^i . \quad (7)$$

In other words, the system is equivalent to m single channel system. In this case, characteristics polynomial of i-th channel is described by polynomial $q_i(s)$, i. e. as $k^{-1} \to 0$ the system becomes autonomous.

Proof.

As to the location of roots of the polynomial $\det A(s)$ one can note the following: equations (42), (43) are equivalent to

$$U(s) = -\mu A^{-1}(s) D^{-1}(s)[-(C(s) + \\ + D(s)R(s)Y(s) + V(s)] .$$

This equation poles are determined by roots of the polynomials $\det A(s)$ and $\det D(s)$. Assume that roots of $\det A(s)$ are located in the closed left half-plane. Poles of $\det D(s)$ we locate in the left half-plane. Actually, the characteristic polynomial for i-th loop $q_i(s)$ is equal to

$$q_i(s) = d_i(s)r_i(s) + c_i(s) ,$$

Where powers of $q_i(s)$ and $d_i(s)$ are equal to $n_i + m_i$ and n_i respectively. Giving $d_i(s)$, we find out uniquely $r_i(s)$ and $c_i(s)$.

Let us prove the Lemma 1. For the sake of simplicity we assume that $n_i = \deg[r_i(s)]$. The substitution of (43) into (39) yields

$$B(s)Y(s) = kZ(s) - kR(s)Y(s) .$$

Now we multiply this expression additionally by $D(s)$ from the left side:

$$[D(s)B(s) + kD(s)R(s)]Y(s) = kC(s)Y(s) + \\ + kV(s) .$$

Substituting here (42), we get

$$[k^{-1}D(s)B(s) + D(s)R(s) + C(s)]Y(s) = V(s) .$$

In that equation all matrices are diagonal ones, therefore it is equivalent to the following system

$$[k^{-1}d_i(s)b_i(s) + d_i(s)s^{n_i} + c_i(s)]y_i(s) = v_i(s) \\ , i = 1,...,m .$$

Power of polynomials $d_i(s)$, $b_i(s)$, $c_i(s)$ are equal to m_i, n_i, $n_i - 1$ respectively. Taking into account (44) as $k \to \infty$ this system can be replaces by the following

$$[d_i(s)s^{n_i} + c_i(s)]y_i(s) = v_i(s) , i = 1,...,m ,$$

or in the compact representation by

$$[D(s)R(s) + C(s)]Y(s) = V(s) ,$$

that is equivalent to (3) taking into account (7).

Thus, for the obtaining the desired dynamics (7) with the channel "i-th input – i-thoutput" one should take regulator of the kind (4), (5). In this case, all the roots of polynomials $q_i(s)$ should be located in the left open half-plane, i. e. their real part must be negative which corresponds to the stable desired dynamics. This condition is sufficient to meet the requirement (6). For the regulator stability one has to require additionally that the roots of the polynomial $\det A(s)$ should be also located in the left half-plane.

Example 1.1.

Let consider the object

$$\begin{bmatrix} \dfrac{k_1}{T_1 s + 1} & \dfrac{k_2}{T_2 s + 1} \\ \dfrac{k_3}{T_3 s + 1} & \dfrac{k_4}{T_4 s + 1} \end{bmatrix} \begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix} = \begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix} .$$

It is required to provide the characteristic polynomial of the first loop equal to

$$q_1(s) = \alpha_3 s^3 + \alpha_2 s^2 + \alpha_1 s + 1 ,$$

and that of the second loop:

$$q_2(s) = \beta_3 s^3 + \beta_2 s^2 + \chi_1 s + 1 ,$$

hence $n_1 + m_1 = 3$, $n_2 + m_2 = 3$.

We transform the object description to the form (6):

$$\begin{bmatrix} T_1 T_2 s^2 + (T_1 + T_2)s + 1 & 0 \\ 0 & T_3 T_4 s^2 + (T_3 + T_4)s + 1 \end{bmatrix} \begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix} = \begin{bmatrix} k_1(T_2 s + 1) & k_2(T_1 s + 1) \\ k_3(T_4 s + 1) & k_4(T_3 s + 1) \end{bmatrix} \begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix} ,$$

$$(T_3 T_4 s^2 + (T_3 + T_4)s + 1)y_2(s) = k_3(T_4 s + 1)u_1(s) + k_4(T_3 s + 1)u_2(s) ,$$

i. e. $n_1 = n_2 = 2$.

Let find out $m_1 = (m_1 + n_1) - n_1 = 1$ and similarly $m = 1$. So, we find out that $\deg[d_i(s)] = 1$, i. e.

$$D(s) = diag\{d_{11}s + d_{10}, d_{21}s + d_{20}\} .$$

Since $\deg[c_i(s)] = n_i - 1 = 1$, then

$$C(s) = diag\{c_{11}s + c_{10}, c_{21}s + c_{20}\} .$$

The only thing left is to write down the matrix

$$R(s) = diag\{s^{n_i}\} = diag\{s^2, s^2\}.$$

According to (45) the desired characteristic polynomial should have the form

$$q_1(s) = s^2(d_{11}s + d_{10}) + (c_{11}s + c_{10}),$$

$$q_2(s) = s^2(d_{21}s + d_{20}) + (c_{21}s + c_{20}),$$

hence, we have elements of matrices $C(s)$, $R(s)$, $D(s)$:

$$d_{11} = \alpha_3, \ d_{10} = \alpha_2, \ c_{11} = \alpha_1, \ c_{10} = \alpha_0 = 1,$$

$$d_{21} = \beta_3, \ d_{20} = \beta_2, \ c_{21} = \beta_1, \ c_{20} = \beta_0 = 1.$$

For the realization of this control law the vectors $y, \dot{y}$ should be accessible. The regulator equation has the form:

$$A(s)U(s) = \mu^{-1}(diag\left\{\frac{q_i(s)}{d_i(s)}\right\}Y(s) +$$

$$+ diag\left\{\frac{1}{d_i(s)}\right\}V(s)).$$

If one takes $m_i = 0$ for each i from (4), (5) regarding that $D(s)$ is a unit matrix we get

$$C(s)Y(s) + Z(s) = V(s), \ (4^*)$$

$$A(s)U(s) = k(Z(s) - R(s)Y(s)). \ (5^*)$$

The equation (45) is transformed as follows

$$q(s) = s^{n_i} + c_i(s), \ (6^*)$$

That is the following corollary is valid.
Corollary 1.

$$U(s) = k\begin{bmatrix} k_1 & k_2 \\ k_3 & k_4 \end{bmatrix}\left(-\begin{bmatrix} s + c_{10} & 0 \\ 0 & s + c_{20} \end{bmatrix}\begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix} + \begin{bmatrix} v_1(s) \\ v_2(s) \end{bmatrix}\right),$$

that is for its realization values $y, \dot{y}$ are necessary.

For some objects the regulator (4^*), (5^*) is physically realizable. As for example, for the objects of the kind $\left[\dfrac{k_{ij}}{1 + T_{ij}s}\right]$ this statement is valid.

Now let us consider the case where the only vector $Y(s)$ is accessible for measurements. Let us construct the regulator of the form

$$D(s)A(s)U(s) = k[-C(s)Y(s) + V(s)], \ (8)$$

where $D(s) = diag\{d_i(s)\}$. The power of polynomials $d_i(s)$ define from the condition

$$\deg[d_i(s)] + \min_j\{\deg[a_{ij}(s)]\} = \deg[c_i(s)], (9)$$

which provides the physical realization possibility of regulator (46). Here, $a_{ij}(s)$ are elements of matrix $A(s)$. Let us denote $\deg[d_i(s)] = r$. The factors of the polynomials $d_i(s)$ will define from

With the growth of k the system (1), (4^*), (5^*) becomes autonomous for output channels. The characteristic polynomial of i-th loop is arbitrarily exactly described by polynomial (7).

Thus, the regulator (4^*), (5^*) provides the desired dynamics of i-th loop of the same order as the power of polynomial $b_i(s)$ (1). The regulator equation can be presented in the explicit form:

$$U(s) = kA^{-1}(s)(-C(s)Y(s) - R(s)Y(s) - V(s))$$

.

Example 1.2.
For the object

$$\begin{bmatrix} \dfrac{k_1}{s} & \dfrac{k_2}{s} \\ \dfrac{k_3}{s} & \dfrac{k_4}{s} \end{bmatrix}\begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix} = \begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix}.$$

it is required to get the characteristic polynomials

$$q_1(s) = s + c_{10}, \ q_2(s) = s + c_{20}.$$

Let the rewrite the object equation in the form (40):

$$\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix}\begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix} = \begin{bmatrix} k_1 & k_2 \\ k_3 & k_4 \end{bmatrix}\begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix},$$

that is $n_1 = n_2 = 1$, $m_1 = m_2 = 1$. Let us make use if Corollary 1. Take $C(s) = diag\{c_{10}, c_{20}\}$, $R(s) = diag\{s, s\}$. Then the regulator equation is

the condition of decoupling of slow and fast motions in i-th loop. In addition, one has to provide the given location of regulator roots and system roots corresponding to motion (for example, they should be in the given sector). Since the characteristic polynomial $d_i(s)$ will be defined from the consideration of

$$\mu d_i(s)b_i(s) + c(s) = 0. \quad (10)$$

where $c_i(s)$ is the desired characteristic polynomial of i-th loop of the power m_i, i. e. $\deg[c_i(s)] = m_i$. Polynomial $b_i(s)$ is included into the object description and its power is n_i. Thus, one should define $d_i(s)$. Let us denote is power as r_i. Denote $q_i = r_i + n_i - m_i - 1$. Take $d_i(s)$ of the kind

$$d_i(s) = \sum_{j=1}^{q_i} \alpha_{i,1+j}\mu^i s^{m_i - n_i + j + 1} +$$

$$+ \sum_{j=0}^{m_i - n_i + 1} \beta_{ij} s^j . \quad (11)$$

Here $\alpha_{i,q_i+1} = 1$, $\beta_{i,m_i-n_i+1} = 1$.

Let us consider the question of choosing the power of polynomial $d_i(s)$ and its coefficients, i. e. r_i, α_{ij}, β_{ij}. Let us denote

$$\min_j \{\deg[a_{ij}(s)]\} = l_i.$$

Then r_i will be taken such as to satisfy the realization

$$r_i \geq m_i - l_i \qquad (12)$$

with its minimal value. As to the choice of coefficients, there are two possible cases. The first case, if $r_i + n_i \leq m_i + 1$, then coefficients are selected from the condition of stability of $d_i(s)$. If $r_i + n_i \leq m_i + 1$, one should rather use the structure of the kind (10). The problem of choice for α_{ij}, β_{ij} is considered in the preceding section. The following lemma is valid.

Lemma 2.

For the object (1) with regulator (12), where the matrix $D(s)$ is determined according to (10) with a decrease of μ the characteristic polynomial of system is determined by the matrix $C(s)$.

Proof.

Substitution (11) into (10) gives:

$$(\mu^{r_i + n_i - m_i} s^{r_i} + \alpha_{i,r_i + n_i - m_i - 1}\mu^{r_i + n_i - m_i - 1} s^{r_i - 1} + ... + \alpha_{i2}\mu^2 s^{m_i - n_i + 2} +$$

$$+ \mu s^{m_i - n_i + 1} + \mu\beta_{i,m_i - n_i} s^{m_i - n_i} + ... + \mu\beta_{i1}s + \mu\beta_{i0})(s^{n_i} + b_{i,n_i-1}s^{n_i - 1} + ...$$

$$+ b_{i0}) + c_{i,m_i} s^{m_i} + ... + c_{i1}s + c_{i0}.$$

After the regrouping the terms taking into account that μ can be done quite small we get:

$$\sum_{j=1}^{r_i + n_i - m_i} \alpha_{ij}\mu^j s^{m_i + j} + \sum_{j=0}^{m_i} c_{ij}s^j .$$

Here $\alpha_{i1} = 0$, $\alpha_{i,r_i + n_i - m_i} = 1$. The slow motions in the obtained relation are defined by the following polynomial

$$c_i(s) = \sum_{j=0}^{m_i} c_{ij}s^j$$

The statement of the lemma is proved.

Thus, for the design of regulator providing the given dynamics $C(s)$ under the assumption of accessibility for measurements of vector $Y(s)$ only in (7) it is necessary to determine $D(s)$ according to (8), (9) and by the method presented in the second section to give the coefficients α_{ij}, β_{ij}.

Example 1.3

For the object given in Example 1.1 it is necessary to design the regulator (45) such that

$$c_1(s) = c_{13}s^3 + c_{12}s^2 + c_{11}s + c_{10},$$

$$c_2(s) = c_{24}s^4 + c_{23}s^3 + c_{22}s^2 + c_{21}s + c_{20}.$$

In the given example we have $n_1 = n_2 = 2$, $l_1 = l_2 = 1$, $m_1 = 3$, $m_2 = 4$,

$$l_i = \min_j\{\deg[a_{ij}(s)]\}.$$

One should choose $r_i \geq m_i - l_i$, hence $r_1 = 2$ (the power of the polynomial $d_1(s)$) and $r_2 = 3$. The check of the relation $r_i + n_i \leq m_{i+1}$ shows that the value of the coefficients of $d_1(s)$ and $d_2(s)$ one has to choose only from the condition of satisfactory form of transient processes in the regulator itself. Thus, according to Lemma 2 we take

$$d_1(s) = s^2 + s\sqrt{2} + 1,$$

$$d_2(s) = s^3 + 2s^2 + 2s + 1.$$

In this example $\deg[a_{ij}(s)] = 1$ for all i, j. It is assumed that the roots of equation $\det A(s) = 0$ are located in the left half-plane.

Corollary 2.

For the object (1) with the regulator (8), where martix $D(s)$ is defined according to

$$\alpha_i(s) = \sum_{i=1}^{n-1} \alpha_{i,1+j}\mu^j s^{j+1} + \beta_{i1}s + \beta_{i0}, \quad (11^*)$$

the characteristic polynomial of i-th loop is described by polynomial $c_i(s)$ of the same power as $b_i(s)$.

That is, for the realization of the characteristic polynomial $c_i(s)$ of power n_i it is necessary to choose $D(s)$ of the form (11*)

Example 1.4

For the object

$$\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix}\begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix} = \begin{bmatrix} k_1 & k_2 \\ k_3 & k_4 \end{bmatrix}\begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix}$$

$$U(s) = k\begin{bmatrix} k_1 & k_2 \\ k_3 & k_4 \end{bmatrix}^{-1}\begin{bmatrix} (s+d_{10})^{-1} & 0 \\ 0 & (s+d_{20})^{-1} \end{bmatrix}\left(-\begin{bmatrix} s+c_{10} & 0 \\ 0 & s+c_{20} \end{bmatrix}X(s)+\begin{bmatrix} c_{10} \\ c_{20} \end{bmatrix}Y(s)\right).$$

2. MULTICHANNEL SYSTEMS: LOWERING OF THE ORDER

Let construct regulator for the object form (11)
$$A(s)U(s) = K[-C(s)Y(s)+V(s)]. \text{ (13)}$$

Here $K(s) = diag\{k_i\}$, $i = 1,2,...n$; k_i are gain coefficients. $C(s) = diag\{c_i(s)\}$, $\deg[c_i(s)] = m_i$.

Let us require that with the growth of K the characteristic polynomial of the system i-th loop is approaching to:

$$N_i(s) = \sum_{j=0}^{m_i} c_{ij}s^j . \qquad (14)$$

Here $m_i < n_i$ (that is just the order lowering). Matrix $C(s)$ is given in the following way:

$$C_i(s) = \sum_{j=1}^{n_i-m_i-1}\alpha_{ij}\mu^j s^{m_i+j} + \sum_{j=0}^{m_i}c_{ij}s^j , \text{ (15)}$$

where n_i is the power of polynomial $b_i(s)$ (compressed into the object), m_i is the power of desired polynomial for i-th loop. Assume that $c_{i,m_i} = 1$, $\alpha_{i1} = 1$. Matrix K will be given as follows

$$K = diag\{k_i\} = diag\{k^{n_i-m_i}\} . \text{ (16)}$$

Lemma 3.

The characteristic polynomial of the i-th loop of system (39), (51) with the growth of k is arbitrarily exactly described by the following expression

$$\sum_{j=1}^{n_i-m_i}\alpha_{ij}\mu^j s^{m_i+j} + \sum_{j=0}^{m_i}c_{ij}s^j . \text{ (17)}$$

Proof.

Under the stability condition (17) with μ small enough we get the necessary property, namely, the system properties are nearing those properties described by (14). For attainment the stability of (17) it is sufficient for the choice of α_{ij} to use the method presented in preceding items.

one has to provide the desired dynamics $s+c_{10}, s+c_{20}$, i. e. $n_1 = n_2 = 1$, $m_1 = m_2 = 0$, $r_1 = r_2 = 1$. According to (11*) we take $d_1(s) = s+d_{10}$, $d_2(s) = s+d_{20}$. That is the regulator has the form

Now we substitute (13) into (1):
$$[K^{-1}B(s)+C(s)]Y(s) = V(s).$$

As K, $B(s)$, $C(s)$ are diagonal matrices, we get that the characteristic polynomial of the system i-th loop is determined by the expression $k_i^{-1}b_i(s)+c_i(s)$. With $k^{-1} = \mu$ taking into account (16) when substituting (15) into presiding relation we get:

$$\mu^{n_i-m_i}(s^{n_i} + b_{i,n_i-1}s^{n_i-1} + ... + b_{i1}s +$$
$$+ b_{i0}) + (\alpha_{i,n_i-m_i-1}\mu^{n_i-m_i-1}s^{n_i-1} +$$
$$+\alpha_{i,n_i-m_i-2}\mu^{n_i-m_i-2}s^{n_i-2} + ...\alpha_{i2}\mu^2 s^{m_i+2} +$$
$$c_{i,m_i}s^{m_i} + ... + c_{i1}s + c_{i0}.$$

After the regrouping terms under the assumption of small value of μ we obtain the lemma statement.

Corollary 3.

The characteristic polynomial of system (1), (13), where
$$K = diag\{k_i\}, k_i = k, \forall i = 1,...,n;$$

$$C(s) = diag\{c_i(s)\}, c_i(s) = \sum_{j=0}^{n_i-1}c_{ij}s^j,$$

$$c_{i,n_i-1} = 1,$$

with the growth of K is arbitrarily exactly described by polynomial $c_i(s)$ for i-th loop.

That is the characteristic polynomial order is reduced by a unity.

The Corollary 3 was obtained from Lemma 3, taking $m_i = n_i -1$, $r_i = r$ in contrast to (16).

Example 2.1.

For two-dimensional object with matrix transfer function $W(s)U(s) = Y(s)$, where $W(s) = [w_{ij}(s)] = [k_{ij}(T_{ij}s+1)^{-1}]$; $i = 1,2$, $j = 1,2$; it is necessary to design regulator with the

desired dynamics by the i-th loop of the kind $s + c_{i0}$.

We reduce the object description to the form (39) and obtain:

$$B(s) = \begin{bmatrix} s^2 + b_{11}s + b_{10} & 0 \\ 0 & s^2 + b_{21}s + b_{20} \end{bmatrix},$$

$$A(s) = \begin{bmatrix} k_1(T_2 s+1) & k_2(T_1 s+1) \\ k_3(T_4 s+1) & k_4(T_3 s+1) \end{bmatrix}.$$

It is assumed that eigenvalues of $A(s)$ belong to the left half-plane. We obtain that $\deg[b_i(s)] = n_i = 2$, $\deg[a_i(s)] = 1$, $m_i = 1$. Therefore, let us make use of Corollary 3: we take $c_i(s) = s + c_{i0}$, $K = diag\{\mu^{-1}\}$. Thus, we get regulator

$$\mu \begin{bmatrix} k_1(T_2 s+1) & k_2(T_1 s+1) \\ k_3(T_4 s+1) & k_4(T_3 s+1) \end{bmatrix} \begin{bmatrix} u_1(s) \\ u_1(s) \end{bmatrix} = - \begin{bmatrix} s + c_{10} & 0 \\ 0 & s + c_{20} \end{bmatrix} \begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix} + \begin{bmatrix} v_1(s) \\ v_2(s) \end{bmatrix}.$$

The characteristic polynomial of i-th loop is equal to $\mu s^2 + s + c_{i0}$. With $\mu \to 0$ we get $s + c_{i0}$, that is required. From the form of regulator equation it follows that for realizing this law it is sufficient to measure vector $Y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}$.

Corollary 4.

For the object (1) with regulator (13), where $\deg[c_i(s)] = \deg[b_i(s)] = n_i$, $k_i = k = \mu^{-1}$,

$$c_i(s) = \sum_{j=0}^{n_i} c_{ij} s^j,$$

the characteristic polynomial of the i-th loop with the growth of k is approaching $c_i(s)$.

In general case, from (17) it is follows that for realizing the control law with desired dynamics one may need the vectors $Y, Y^{(1)}, ... Y^{(r)}$, where

$$r = \max\{r_i\},$$

$$r_i = \deg[c_i(s)] = \min_j \{\deg[d_j(s)]\}.$$

Let us consider the construction of regulator providing the desired dynamics of reduced order with accessibility for measurements only vector Y. Let us choose the regulator form as

$$D(s)A(s)U(s) = K[-C(s)Y(s) + V(s)], \text{ (18)}$$

Here $D(s) = diag\{d_i(s)\}$, $K = diag\{k_i\}$, $C(s) = diag\{c_i(s)\}$, $i = 1,2,...m$. We take $\deg[c_i(s)] = n_i - 1$, where $n_i = \deg[b_i(s)]$. Let denote $\deg[d_i(s)] = q_i$. Dimensions of the matrices $A(s)$, K, $C(s)$ are $n \times n$. From the possibility of realization of regulator (18) we impose the limitation

$$\sum_{j=1}^{m_i} d_i(s)a_{ij}(s)u_j(s) = -k_i c_i(s) y_i(s) + v_i(s),$$

i. e. $q_i + l_i \geq n_i - 1$ for all i, where $l_i = \min_j \{\deg[a_{ij}(s)]\}$. We take minimal order $q_i = n_i - l_i - 1$ and give $d_i(s)$ and $c_i(s)$ as followings:

$$d_i(s) = \sum_{j=0}^{q_i} \alpha_{i,n_i - m_i + j} \mu^j s^j, \text{ (19)}$$

$$c_i(s) = \sum_{j=1}^{n_i - m_i - 1} \alpha_{ij} \mu^j s^{m_i + j} + \sum_{j=0}^{m_i} c_{ij} s^j, \text{ (20)}$$

where $\alpha_{ij} = 1$, $\alpha_{i,n_i - m_i + q_i} = 1$, $c_{i,m_i} = 1$,

$$k_i^{-1} = \mu^{n_i - m_i}.$$

Lemma 4.

For the object (1) with regulator (18) with the growth of k_i and with the choice of $d_i(s)$ and $c_i(s)$ according (19) and (20), the characteristic polynomial approaches the form

$$\sum_{j=1}^{n_i - m_i + q_i} \alpha_{ij} \mu^j s^{m_i + j} + \sum_{j=0}^{m_i} c_{ij} s^j, \text{ (21)}$$

where $\alpha_{ij} = 1$, $\alpha_{i,n_i - m_i + q_i} = 1$, $c_{i,m_i} = 1$.

If the polynomial remains stable at $\mu \to 0$, the first term of (21) determines the fast roots and the second one determines the slow roots. Thus, in this lemma it is stated that slow motions in the system are determined by polynomial (14). Here $m_i < n_i$,

$$\sum_{j=1}^{n_i-m_i+1} \alpha_{ij}\mu^j s^{m_i+j} + \mu \sum_{j=0}^{q_i} \mu^j \alpha_{i,n_i-m_i+j} s^j + c_i(s) .$$

If we remove the parenthesis, rake into account that $\mu^i >> \mu^{i+1}$, group the similar terms, separate terms containing μ in minimal powers and neglect all the rest, we get the equation (20).

One can formulate the corollary.

Let us consider the special case. Let choose the following structure of regulator

$$c_i(s) = \sum_{j=0}^{n_i-1} c_{ij}s^j , \; c_{i,n_i-1} = 1 ,$$

$$d_i(s) = \sum_{j=0}^{q_i} \alpha_{i,j+1}\mu^j s^j , \; \alpha_{i1} = 1 ,$$

$$\alpha_{i,q_i+1} = 1 , \; k_i^{-1} = \mu ,$$

$$q_i = n_i - l_i - 1 . \text{(60)}$$

Corollary 5.

For he object (1) with regulator (18) with the choice of $d_i(s)$ and $c_i(s)$ by (22), at $\mu \to 0$ the characteristic polynomial of i-th loop is approaching the form

$$\sum_{j=1}^{q_i} \alpha_{ij}\mu^j s^{n_i+j} + \sum_{j=0}^{n_{-1i}} c_{ij}s^j . \text{ (23)}$$

The Corollary 5 can be interpreted as follows. If the required dynamics of the i-th loop should have

m_i is the power of the desired characteristic polynomial of the i-th loop.

Let us prove the Lemma 4. Substituting (18) – (21) into (1) one gets the system characteristic polynomial of the form

the order $n_i - 1$, then for the regulator design one should use formulae (18) and (22). For proving the Corollary 5, it is sufficient to give $m_i = n_i - 1$ in Lemma 3.

Example 2.2.

Let the object be described by the matrix transfer function [10]:

$$W(s) = \begin{bmatrix} \dfrac{s+2}{s^2+4s+3} & \dfrac{1}{s^2+4s+3} \\ -\dfrac{s+1}{s^2+4s+3} & \dfrac{1}{s^2+4s+3} \end{bmatrix} .$$

It is required to design the regulator providing the desired dynamics by the first and second loops, respectively:

$$N_1(s) = s + c_{10} , \; N_2(s) = s + c_{20} .$$

Here

$$A(s) = \begin{bmatrix} s+2 & 1 \\ -(s+1) & s+1 \end{bmatrix} ,$$

$$B(s) = \begin{bmatrix} s^2+4s+3 & 0 \\ 0 & s^2+4+3 \end{bmatrix} ,$$

i. e. $\deg[b_i(s)] = 2$, $m_i = 1$. As $l_i = 1$, $q_i = 0$, then $d_i(s) = 1$, i.e. $D(s)$ is unit matrix, $K^{-1} = diag\{\mu\}$. Thus we obtain the regulator in the form

$$\mu \begin{bmatrix} s^2+2 & 1 \\ -(s+1) & s+1 \end{bmatrix} \begin{bmatrix} u_1(s) \\ u_1(s) \end{bmatrix} = -\begin{bmatrix} s+c_{10} & 0 \\ 0 & s+c_{20} \end{bmatrix} \begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix} + \begin{bmatrix} v_1(s) \\ v_2(s) \end{bmatrix} .$$

The task is solved.

CONCLUSION

In the paper the development of method of the dividing of motions has been developed. The results are proven by mathematical conversion taking into account that it is allowed to neglect small terms in equations comparatively much bigger terms with

the same power of argument s (argument of Laplace transform which is relative to frequency). The proposed method does not allow getting of astatic multi-channel system. For this purposes it is necessary to use preliminary integrator in the input of the channel, as it is recommended for SISO systems in paper [2].

In this case the integrator transfer function in multi-channel case has the following form:

$$W_{Int}(s) = diag[\frac{1}{s}] = \begin{bmatrix} \dfrac{1}{s} & ... & 0 \\ ... & ... & ... \\ 0 & ... & \dfrac{1}{s} \end{bmatrix}.$$

This multiplier is set at the object input before the procedure of the regulator design. It is essentially, that in this case the order of the resulting new object is increasing.

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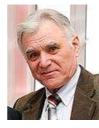
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