Control of Linear Dynamic Objects by the Method of Division of Motions

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Abstract: **In this paper, the problem of the design of control law for linear object with time variable parameters by use of full or truncated state vector of the object is considered. The method is based on the use of state vector, motions devising principle and localization of disturbances. It provides the system dynamic features to be in accordance with the differential equations of the given order with the accuracy to neglecting of the fast processes which are insignificant in compare with output signals. In this case, the system equation can be done lower of higher then the order of the object equation.**

Key words: **division motions method, localization method, feedback, control, automatics, dynamic error, static error**

INTRODUCTION

The problem of design of system with nonlinear nonstationary objects or linear objects with inexactly known parameters is rather important due to the development of control methods for complex dynamic objects [1–38]. Such situation occurs in the problems of control of movable objects when the control systems for various technological processes are produced. The methods developed by now for design of control algorithms do not always satisfy the requirements of the up-to-date practice of design. This especially concerns to nonlinear objects with the features of parameters whose rate of variations is comparable with the rate of transient processes.

Let consider the problem of the regulator design for the system of automatic control. The system consists of regulator, object and sensor of the output value. Its equations are:

$$u = r(x, y, v)\,,\; x = \theta(u)\,,\; y = p(x)\,.$$

Here r, θ, y are operators describing the regulator, object and sensor respectively; *x*, *y*, *v* and *u* are the state, output value, prescription and control vectors. The sensor *p* shows functional dependence of the state vector on the vector of output values being always acceptable for measurements. The regulator *r* is arranging the control in function of *x* if *x* is accessible for measurement or for function *y* in the inverse case in such a way that the following property takes place:

$$\lim_{t\to\infty} y(t) = y_0(t)\,.$$

Here $y_0(t)$ is prescribed value of the vector of output value determined by the vector *v* and the system of motion equations. The limiting relation here means that for any $\varepsilon > 0$ there is $T > 0$ such, that $\| y(t) - y_0(t) \| < \varepsilon$ for any $t \ge T$.

We shall consider the control of object whose behavior model in general case has the following form:

$$\dot{x} = f(t, x, u)\,,\; y = g(x)\,,\; x \in R^n\,,$$

$$u, y \in R^m\,.$$

Here *f*, *g*, are some functions (in our case, linear ones). The problem of design consists in the search for such operarot *r* that the system would satisfy some requirements. As a rule, initial requirements to dynamic properties of the system are formulated in terms of estimates of transient processes either in time field or in some cases they are given by some criterion of the processes optimum. With the use of some certain method of design we go from estimate to one of forms of system dynamic characteristics in a complex. For example, at the design of system by use of frequency characteristics we go from such characteristics as speed and overshooting to the desirable characteristic s of the unlocked or locked system. At the design with the use of root methods one can go to the desirable positioning of roots of the system characteristic equation. The processes in the system can be wholly given by the following ways:

- to provide the desirable differential equation of the system which identically determines all processed;
- to provide the given totality of transient processes;
- to form the given totality of transient processes;
- to satisfy some optimum criterion along transient processes.

Thus, in general case, one needs the transition step from initial estimates of the transient processes to one of the given above representations of full former of dynamic properties. The second step is totally determined by actual technical task, and it is difficult to given some general recommendations, therefore further we shall assume that the design differential equations are given as desirable dynamic properties. We assume that differential equations of the system are written in the form

$$\dot{x} = F(x, v)\;.$$

Here *F* is function built or chosen on the base of the required estimates of transient processes. Further we shall call this equation “desirable” (required, given). Thus, the purpose of the design is to find out such a control low $u = r(t, x, v)$ (the explicit time dependence is possible) which could provide the object motion $\dot{x} = f(t,x,u)$ according to the given equation $\dot{x} = F(x,v)$. If one could observe only the output vector *y*, one needs a special device for evaluating *x*. We have discussed the content of design problem and gave the formulation of the problem of control. Further let us consider some possible approaches to its solution. The main and mostly advanced structure of systems for automatic control assumes the use of inconsistency $\varepsilon = v - y$, i. e. $u = r(\varepsilon)$. Here and further we take the vectors *v* and *y* are concerted in dimensions. The purpose of functioning of such a system is to suppress initial deviation $\varepsilon(0)$ and, quite naturally, the use of ε as an initial value for the regulator forming the controlling action to the object. With either nonlinear objects or those with variable parameters the control low as a deviation function in general case, is not able to provide the invariance of the development processes.

If at the design of nonlinear systems of systems with variable parameters one would use as a basis today developed methods of correction and design, this would lead to adaptive systems with identification. The schematic of such a system is the following:

$$x = \vartheta(u,a),\ u = r(v,\hat{a}),\ \hat{a} = i(u,x).$$

Here *a* is the object parameter to be evaluated, $\hat{a}$ is its estimate, *i* is operator identifying the object parameter "*a*" by signal *u*, *x*. In general case, this problem is quite complex and in its solution they usually try to go to the regulator tuning for one or two general parameters.

More developed class of adaptive systems is the standard model systems. There is a quite well developed theory for building of searchless self-tuning systems based on the use of standard model one of whose version is the following:

$$u = r(v,t,x,\hat{a}),\ x = \vartheta(u),\ \hat{a} = l(x - x_m),$$

$$x_m = m(v).$$

Here *m* is model, x_m is model output, *l* is adapting device. The difficulties in realization the adaptive control low are the same as those for adaptive systems with identification, though the choice and realization of adaptation algorithms by now are well developed and put in the base of many projects with the time parameter variations relatively low compared to the rate of transient processes of the main system.

An efficient way to control the complex dynamic objects incorporated the best features of all above mentioned methods might be the use in the control law of the highest derivative of output value or in the vector case, the velocity vector of state coordinates. Feedback loops of such kind found their use in the development of system quite recently. The report by Grin [1] is apparently one of the first publications on the subject. Starting from this work in methods of control for flying machines such a method is used regularly, for example, manuscript by A.P. Batenko [2]. The idea of such an approach was formulated in the paper by G.S. Pospelov [3]. The possibilities of the use of feedback with the highest derivative value were discussed by L.M. Boichuk [4] and A.S. Vostrikov [5, 6]. From theoretical viewpoint the possibilities of this approach are quite large which can easily be shown by an example of the scalar object [7]. In this case, the object is described by the following differential equation

$$x^{(n)} = f(t,x,...x^{(n-1)}) + b(t,x,...x^{(n-1)})u. \quad (1)$$

Here *x* is an output value, $x^{(i)}$ is its *l*-th derivative, us is a controlling action, *f* and *b* are functions depending on time and states $x,...x^{(n-1)}$. The purpose of regulation consists in:

- organization of static property

$$\lim_{t\to\infty} x(t) = \lim_{t\to\infty} v(t)$$

where *v*(*t*) is prescription;

- desired dynamic properties of the system with regulator are given with standard differential equation

$$x^{(n)} = F(t,x,...x^{(n-1)},v). \quad (2)$$

Here *F* is linear stationary function corresponding to the required dynamic properties and the regulation purpose in the static. i. e. *F* = 0 at *x*(*t*) = *v*(*t*).

The key idea of the most efficient method, called as the method of division of motions, providing the localization of disturbances consists in the organization of the inner feedback loop circuit where nonstationary actions are localized. For this aim, we select the regulator of the form [7]:

$$u = k[F(t,x,...x^{(n-1)},v) - x^{(n)}].$$

Here *k* is an gain coefficient. When substituting this expression into the object equation (1) we get the system of the form

$$x^{(n)} = F + \frac{F - f}{1 + bk}. \quad (3)$$

If the value *bk* is selected large enough due to *k*, the second summand in the fight hand side decreases. In the limit at $k \to \infty$ one can get an equation $x^{(n)} = F$. In practice, in view of required level of approximation *k* is usually selected to be equal to *k* = 20…100.

The controlling action remains finite at any value of *k*:

$$\lim_{k\to\infty} u = \lim_{k\to\infty} \frac{k(F - f)}{1 + bk} = \frac{F - f}{b}. \quad (4)$$

The system structure (*Fig*. 1) is described by the following equations

$$\hat{x}^{(n)} = F(v,x),\ \varepsilon = \hat{x}^{(n)} - x^{(n)},\ u = k\varepsilon, \quad (5)$$

$$x^{(n)} = f(t,x,...x^{(n-1)}) + b(t,x,...x^{(n-1)})u. \quad (6)$$

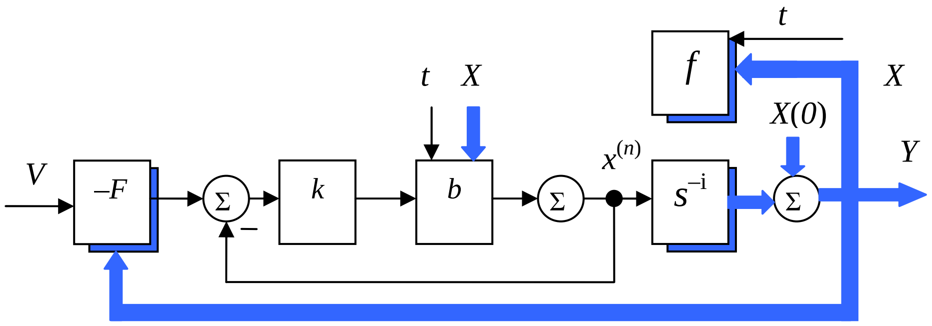

Fig. 1. Block diagram of the system with regulator by the method of division of motions

It is seen that together with the common loop $x \to \hat{x}^{(n)} \to u \to x$ the inner loop is built where the object instability influence is localized: $\varepsilon \to u \to x^{(n)} \to \varepsilon$.

In the work presented here the method is proposed which enables one both to raise and to reduce the system order compared to that of the object. The procedure of regulator design provides the stability of the control inner loop at any either finite or infinite value of *k* and under the condition that the value is always larger then any parameter of the object. In this case, the approximation of the system differential equation to that desired is the more exact the larger is *k*. This method is also extended to multivariable systems. i. e. the systems where the input and output variables are vector values of finite dimension. In the work the control design problem is considered both with the complete data on the object conditions and by the output signal *y* only.

The method of design of regulator for nonlinear systems using the state vector, motion division principle and localization of disturbances was primarily proposed in the works of A.S. Vostrikov [7–9] for the case when the desired equation of the system coincides with the order of object equation. The problems of the design of control systems where considered in Refs. [10–38]. The method of division of motions is considered in section 1 in more details.

1. FOUNDATIONS OF METHOD OF DIVISION OF MOTIONS

Let us consider the case where it is necessary to control the object whose parameters are either known insufficiently accurately or time variable. It is required that the system behavior should be described by differential equations of the given kind or, in other words, the characteristic polynomial of the locked system should have the given form with the demanded coefficients. In more details, let the object transient response be shown in *Fig*. 2 (line 1), and the possible curves for processes determined by the drift of parameters are shown by other lines. Let us assume that from the engineering viewpoint we would like to built the control system in such a way to get the transient process of given form at the stepwise action to the system as, for example, shown in *Fig*. 3.

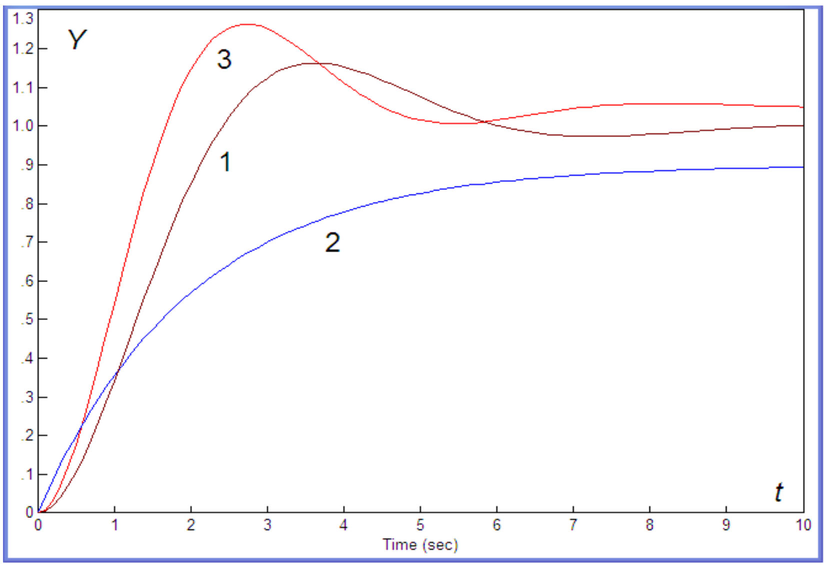

Fig. 2. The object transient responses due to the drift of its parameters

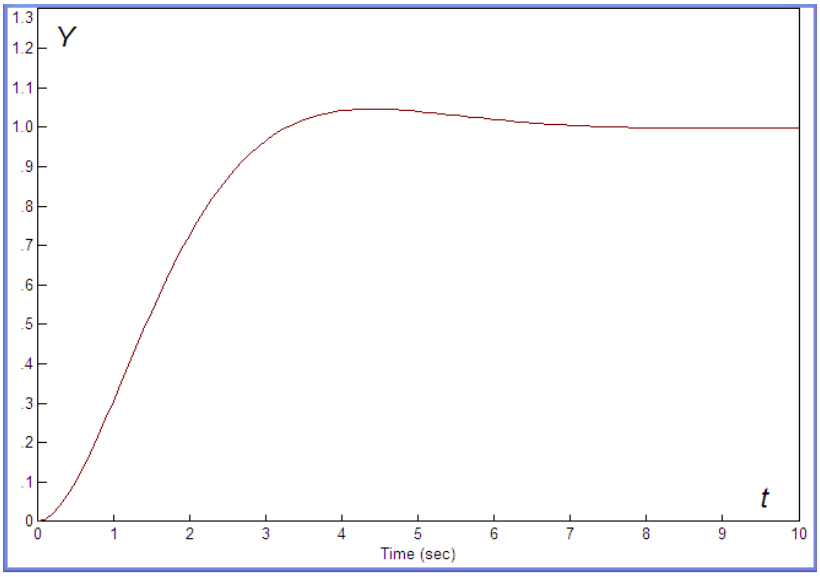

Fig. 3. The example of the desired transient responses

This corresponds to the requirement to the system "object + regulator" to be describes by differential equation

$$T_1T_2\ddot{y}+(T_1+T_2)\dot{y}+y=ku$$

which can easily be found in reference books or in the form of transfer function

$$k(T_1T_2s^2+(T_1+T_2)s+1)^{-1}$$

i.e. one can assume that the object is given (*Fig.* 4) whose parameters we know insufficiently accurate of the parameters vary with time.

The following designations are introduced here: u is control value, y is output value, f is a disturbance. It is necessary to develop the control device (*Fig.* 5) where v is prescribed value acting such as to provide the given form of characteristic polynomial. But practice as always puts its corrections. In this case, the following considerations could be of such a correction. As is well known, for the regulator operation one needs to have either the knowledge of all "internal" variables (called the state vector) and the problem can exactly be solved or they are evaluated approximately with the help of observers and differentiators but here the problem is solved approximately. In this statement the problem is quite complex and frequently it is unsolvable. The solution methods are of great variety. In our case, we consider that only an output value can be measured and therefore for the lack of information we shall pay with that in some part the diagram of transient process will be slightly differ from that desired.

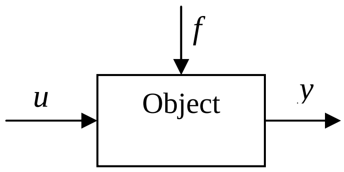

Fig. 4. Controlled object

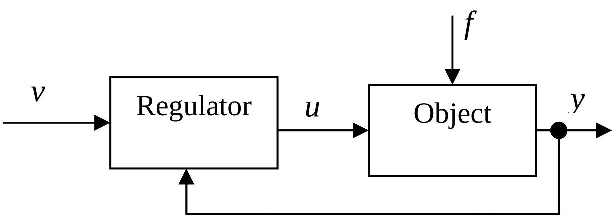

Fig. 5. Controlling system

Let us consider the essence of the method of division of motions proposed in Ref. [5–7]. Let us try to build the system in such a way that all the deviations of parameters and disturbances are worked out in the feedback loop specially formed in the system and the required form of transient process, whose example is given in Fig. 3, would provide by another loop. Let us call them the fast and slow loops respectively. By this way we perform the division of motions (*Fig.* 6), i. e. the motions as they were separated into the fast and slow ones, and they are performed in the fast and slow feedback loops respectively.

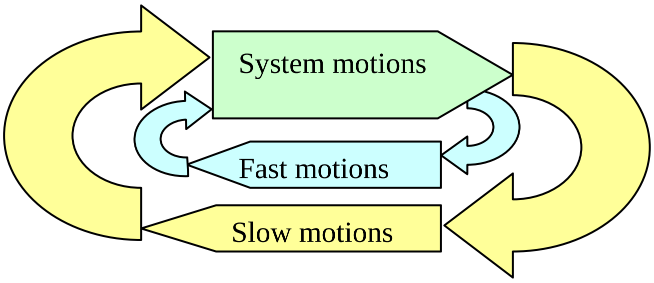

Fig. 6. Dividing of motions in the system

If the motions in fast loop are performed much faster that those in the slow loop then it is likely localizes all disturbances and in the loop of slow motions it is hardly noticeable.

As was said above, the regulator is taken in such a way to get explicitly or implicitly two control loops (*Fig*. 7). The control device includes three blocks: Regulator 1 (R1), Regulator 2 (R2) and derivative filter (DF). The later plays role of observing device (in our case the output signal y is acceptable and it is necessary to evaluate somehow the state vector). Motions y_l performed along the inner loop $R2 \to Object \to DF$ are much faster than these y_s along the outer loop $R1 \to R2 \to Object$ (Fig. 7). With the correct construction of the system fast motions are damped fast and they are negligible for the system output. They are only of a few percents by amplitude.

From the mathematical point of view it can be presented in the following way. The system properties are described by the characteristic polynomial. For each its root there is corresponding mode of transient process. Consequently there two groups of roots corresponding to the fast and slow motions. The roots comparable by modulus to unity – to slow motions (*Fig*. 9).

Thus, the characteristic polynomial of closed system (*Fig*. 7) can be written in the form $a(s)\cdot b(s)$, where the polynomial $a(s)$ corresponds to fast motions, $b(s)$ – to slow ones. Let us formulate the design problem for control device (regulator): it should be such a device that the characteristic polynomial of the locked system would include two multipliers, $a(s)$ and $b(s)$, where $a(s)$ determines fast motions and $b(s)$ – slow ones. The required quantity both for fast and slow motions will be proves by the location of roots in the given region (*Fig*. 9).

In the considered structure of regulator for the both loops the signal is taken from the object output and after the passing it through the control device it reaches the object input, i. e. with the help of structure given in Fig. 7 to that in Fig. 10. We have combines the loop of fast motions with the loop of slow ones. Nevertheless, these motions are divided into fast and slow motions and though conventionally but the fast loop is present. Such a transformation simplifies the realization of the regulator and simplifies the design of it a little.

The above method can be extended to discrete systems. Because of limiting space of the publication further we consider only continuous systems. The solution existence for the problem is illustrated by two examples.

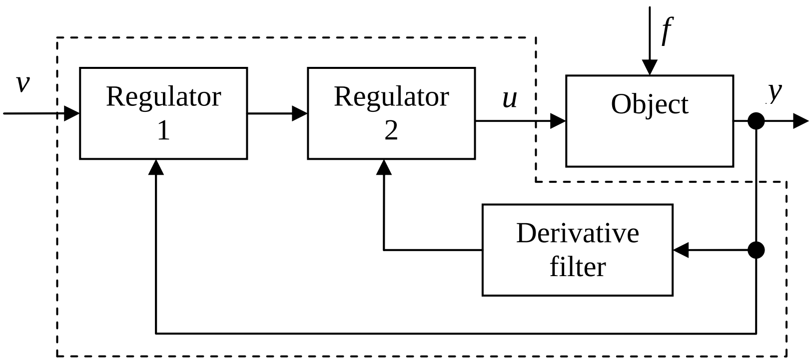

Fig. 7. Structure if regulator with separated loops

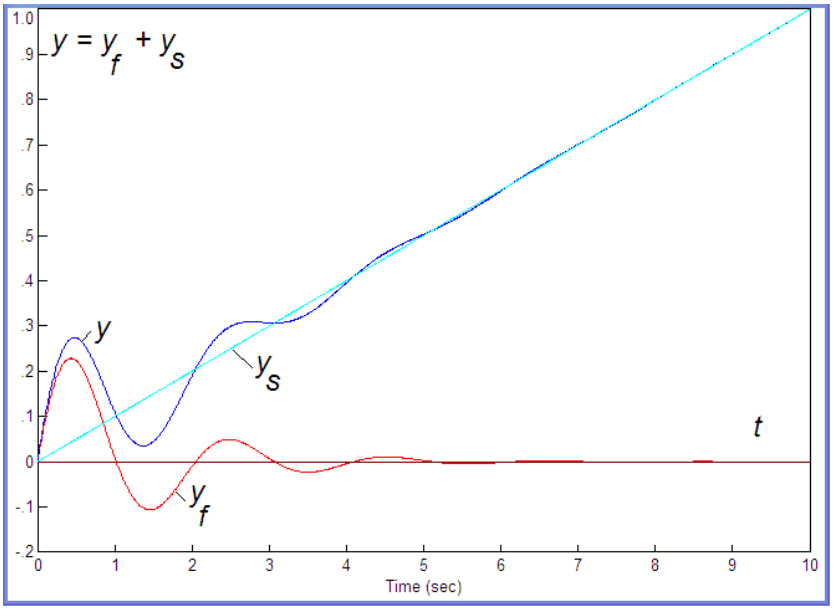

Fig. 8. Fast and slow motions

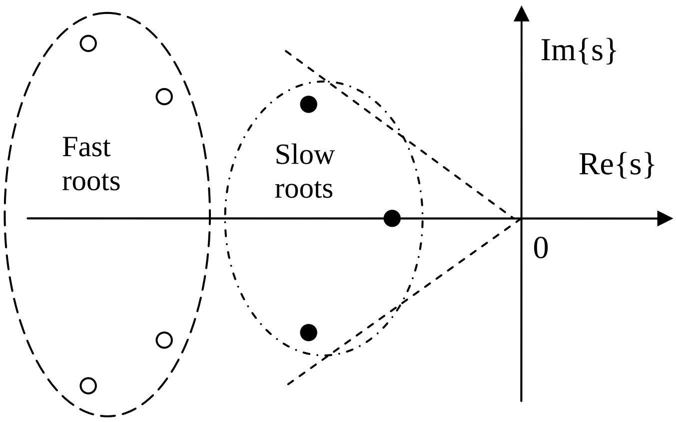

Fig. 9. The two groups of roots in the division of motions method

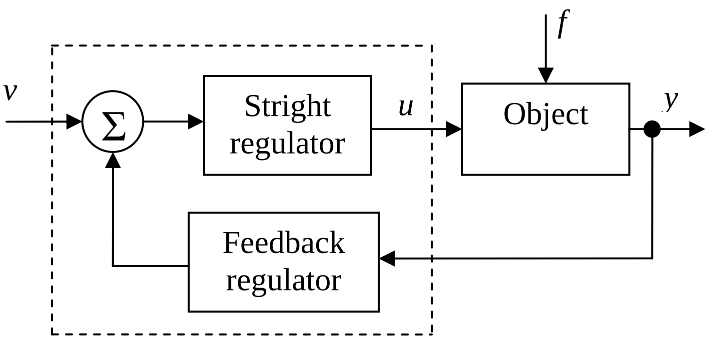

Fig. 10. System structure

Example 1.1.

Let it be given object described by the second order equation:

$$\ddot{y} + a_1\dot{y} + a_0 y = bu\,. \qquad (7)$$

It is equivalent to the transient function:

$$W_O(s) = \frac{b}{s^2 + a_1 s + a_0}\,. \qquad (8)$$

We need to construct the control system such as the transfer processes in the system would be desired by the third order equation:

$$(c_3 s^3 + c_2 s^2 + c_1 s + 1)y = v\,. \qquad (9)$$

In this case it is assumed that if the system fast motions are occurred, their influence on processes in the system is negligible and they are not described by previous equations.

That is, strictly speaking, the previous equation describes slow motions and consequently when taking of the desired dynamics, we mean allow motions. Let us take the regulator of the kind as follows

$$u = k\frac{c_3 s^3 + c_2 s^2 + c_1 s + 1}{b(\mu s^3 + s^2 + \alpha s + 1)} x . \qquad (10)$$

Here k/b determines the regulator gain, μ is small number (parameter), selected usually to be smaller than c_i. The quality of regulator output process (diagram form $u(t)$) depends on value α. The system structure diagram is given in *Fig.* 11, where one can see that for restoring the state vector the differential section (unit in feedback loop) are used. One can also separate loops of fast and slow motions. For that it is sufficient to cut the section of back coupling feedback loops into two sections connected in series:

$$W_{01}(s) = 1,\ W_{02}(s) = c_3 s^3 + c_2 s^2 + c_1 s + 1.$$

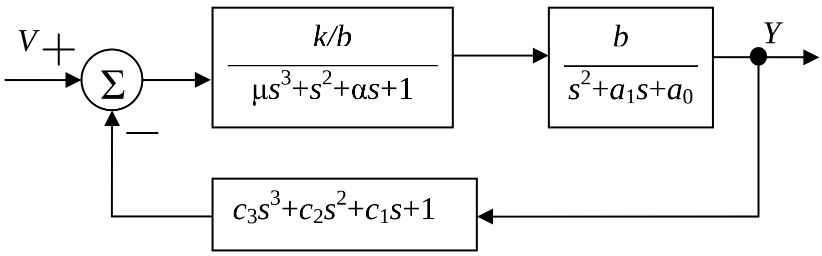

Fig. 11. System structure according Example 1.1

Let us find out the transfer function of control system:

$$W_C(s) = W_1(s)(1 + W_1(s)W_2(s))^{-1},$$

where $W_1(s)$ is the transfer function of direct branch of the loop, $W_2(s)$ is the transfer function in feedback loop, $W_C(s)$ is the transfer function of locked system. Namely, we will write down the object and regulator equations in the form

$$(s_2 + a_1 s + a_0) y(s) = bu(s), \qquad (11)$$

$$b(\mu s^3 + s^2 + \alpha s + 1)u(s) = -k(c_3 s^3 + c_2 s^2 + c_1 s + 1)y(s) + kv(s). \qquad (12)$$

Let us cancel intermediate variable u(s) from two equations. Let multiply both the left and the right sides of equation (11) by the factor

$$(\mu s^3 + s^2 + \alpha s + 1).$$

It gives

$$(\mu s^3 + s^2 + \alpha s + 1)(s^2 + a_1 s + a_0)y(s) = b(\mu s^3 + s^2 + \alpha s + 1)u(s). \qquad (13)$$

It is seen that the right side of (13) considers with the left-hand side of (12). Hence

$$(\mu s^3 + s^2 + \alpha s + 1)(s^2 + a_1 s + a_0)y(s) = -k(c_3 s^3 + c_2 s^2 + c_1 s + 1)y(s) + ku(s). \qquad (14)$$

Thus, from two equations (11) and (12) describing the system shown in *Fig.* 11, we cancelled intermediate variable $u(s)$ and obtained the relation between the systems input values. We can obtain now the system transfer function. But for this purpose, we should preliminary group the terms containing variable $y(s)$. Let us open the parenthesis in the left-hand side of (14):

$$(\mu s^5 + (\mu a_1 + 1)s^4 + (\mu a_0 + a_1 + \alpha)s^3 + (a_0 + a_1\alpha + 1)s^2 + (\alpha a_0 + a_1)s + a_0)y(s) = k(c_3 s^3 + c_2 s^2 + c_1 s + 1)y(s) + kv(s).$$

Now it is necessary to group the terms containing $y(s)$. Let us take k to be equal to μ^{-1}, then

$$[\mu^2 s^5 + (\mu^2 a_1 + \mu)s^4 + (\mu^2 a_0 + \mu a_1 + \mu\alpha + c_3)s^3 + (\mu a_0 + \mu a_1\alpha + \mu + c_2)s^2 + (\mu\alpha a_0 + \mu a_1 + c_1)s + (\mu a_0 + 1)]y(s) = v(s). \qquad (15)$$

Let us select μ to be small enough, then

$$\mu^2 a_1 + \mu \approx \mu,$$
$$\mu^2 a_0 + \mu a_1 + \mu\alpha + c_3 \approx c_3,$$
$$\mu a_0 + \mu a_1\alpha + \mu + c_2 \approx c_2,$$
$$\mu\alpha a_0 + \mu a_1 + c_1 \approx c_1,\ \mu a_0 + 1 \approx 1.$$

Then the equation (15) with sufficient level of accuracy can be replaced by

$$(\mu^2 s^5 + \mu^2 a_1 s^4 + c_3 s^3 + c_2 s^2 + c_1 s + 1)y(s) = v(s). \qquad (16)$$

Let us call it asymptotic equation. The letter equation describes the system the more exactly, the smaller is μ. With an appropriate choice of μ one can achieve that factors of (15) and (16) could differ from each other no more than the preliminary given value.

But the polynomial roots are always dependents of its coefficients. Therefore, by the selection of μ (increasing k) is the system fast action in increased and disturbances are compensated much faster.

Let us consider an asymptotic characteristic polynomial of the system given in *Fig*. 11. Than is a polynomial of the left-hand side of equation (16). It can be as follows

$$c_3^{-1}(\mu^2 s^2+\mu s+c_3)(c_3 s^3+c_2 s^2+ +c_1 s+1). \quad (17)$$

Actually, let us open parentheses:

$$\mu^2 s^5+(\mu+c_2 c_3^{-1}\mu^2)s^4+(c_3+c_2 c_3^{-1}\mu+ +c_1 c_3^{-1}\mu^2)s^3+(c_2+c_1 c_3^{-1}\mu+c_3^{-1}\mu^2)s^2+ +(c_1+\mu c_3^{-1})s+1\approx\mu^2 s^5+\mu^2 s^4+ +c_3 s^3+c_2 s^2+c_1 s+1.$$

Let us consider polynomials in (17). The roots of polynomial $\mu^2 s^2+\mu s+c_3$ by modulus are large enough and the smaller is μ, the farther they are located from the origin of the coordinates and is, in addition, they are located in the left-hand half of the plane (that is so if $c_3>0$) so the faster damps the component of transient process corresponding to the polynomial roots. The second multiplier $c_3 s^3+c_2 s^2+c_1 s+1$ corresponds to slow motions. Thus, it is clear that it correspond to slow motions. Thus, it is clear that the problem of increasing the order has a solution in the class of continuous systems and it is considered in second and forth sections.

Example 1.2.

Let is consider the case of decreasing the order of desired dynamics. Let the object is described by the third order equation:

$$u(s)=b(s^3+a_2 s^2+a_1 s+a_0)^{-1}u(s).$$

We take the regulator

$$u(s)=b^{-1}\mu^{-2}(\mu^2\alpha_4 s^2+\mu\alpha_3 s+\alpha_2)^{-1}[v(s)- -(\mu\alpha_1 s^2+c_1 s+1)y(s)].$$

Let us find out the system equation. Rewrite equation:

$$(s^3+a_2 s^2+a_1 s+a_0)y(s)=bu(s),$$

$$\mu^2 b(\mu^2\alpha_4 s^2+\mu\alpha_3 s+\alpha_2)u(s)=-(\mu\alpha_1 s^2+ +c_1 s+1)y(s)+v(s)$$

Let multiply both the left and the right side of the first equation by

$$\mu^2(\mu^2\alpha_4 s^2+\mu\alpha_3 s+\alpha_2).$$

Then the right side of the first equation can be substituted by the right side of the second equation:

$$\mu^2(\mu^2\alpha_4 s^2+\mu\alpha_3 s+\alpha_2)(s^3+a_2 s^2+ +a_1 s+a_0)y(s)= =-(\mu\alpha_1 s^2+c_1 s+1)y(s)+v(s).$$

Let us multiply polynomials in the left side.

We get

$$\mu^2\alpha_4 s^5+(\mu\alpha_3+\mu^2\alpha_4 a_2)s^4+(\alpha_2+ +\mu\alpha_3 a_2+\mu^2\alpha_4 a_1)s^3+ +(\alpha_2 a_2+\mu\alpha_3 a_1+\mu^2\alpha_4 a_0)s^2+ +(\alpha_2 a_1+\mu\alpha_3 a_0)s+\alpha_2 a_0.$$

Hence the system equation is

$$[\mu^4\alpha_4 s^5+(\mu^3\alpha_3+\mu^4\alpha_4 a_2)s^4+ +(\mu^2\alpha_2+\mu^3\alpha_3 a_2+\mu^4\alpha_4 a_1)s^3+ +(\alpha_2 a_2\mu^2+\mu^3\alpha_3 a_1+\mu^4\alpha_4 a_0+\alpha_1\mu)s^2+ +(\mu^2\alpha_2 a_1+\mu^3\alpha_2 a_0+c_1)s+1]y(s)=v(s).$$

The asymptotic polynomial has the form

$$\mu^4\alpha_4 s^5+\mu^3\alpha_3 s^4+\mu^2\alpha_2 s^3+\mu\alpha_1 s^2+c_1 s+1.$$

If the characteristic polynomial is presented in the form of a product of two polynomials, one can get that the fast motions are described by the fourth-order polynomial (mnemonic rule is to through out the terms containing μ and one gets polynomial describing slow motions $c_1 s+1$). Consequently for fast motions the fourth power of s remains, i. e.:

$$c_1^{-1}(\mu^4\alpha_4 s^4+\mu^3\alpha_3 s^3+\mu^2\alpha_2 s^2+\mu\alpha_1 s+c_1)$$

Thus, allow motions are described by characteristic polynomial of the first order with the third order objects. So the problem of decreasing the order has solutions (Sections 3 and 5).

From the above examples such advantages of the method of devising of motions follows as:

- independence of the desired equation order of the object order;
- the possibility if an arbitrary exact approximation to the desired equation within the admissible restrictions for input actions on the object;
- possibility of physical realization of regulator (there is no need in ideal differentiation);
- reduced dependence of system stability on the object parameters is achieved.

2. SYSTEM WITH SINGLE INPUT AND SINGLE OUTPUT (SISO): ORDER CREATING REGULATOR

Let the object be described by the following equation

$$\sum_{i=0}^{n} a_i s^i x(s)=bu(s), \quad (18)$$

where $x(s)=L\{x(t)\}$ is Laplace transform from $x(t)$, $a_n=1$, $u(s)=L\{u(t)\}$. Let us call the stable polynomial of n-th power the expression of the form

$$N_n(s)=\sum_{i=0}^{n} c_i s^i, \quad (19)$$

if the part of all roots of equation $N_n(s)=0$ is negative. Let vector $x(t),x^{(1)}(t),\dots x^{(n)}(t)$ be acceptable for measurements. Let us construct a regulator:

$$\sum_{i=0}^{m} c_{n+i} z^{(i)} + \sum_{i=0}^{n-1} x^{(i)} = v\,,\ c_{n+m} = 1,\ m > 0\,,$$

$$u = k(z - x^{(n)})\,, \quad (20)$$

where c_i is the factor of polynomial (19), $v(t)$ is prescription value, k is a gain.

Lemma 1. Characteristic equation of object (18) with regulator (20) with an increase of k can be arbitrary exactly approached to the equation

$$N_{n+m}(s) = 0\,,\ c_{n+m} = 1. \quad (21)$$

Proof. From (18) and (20) we get:

$$[(1+bk)s^n + \sum_{i=0}^{n-1} a_i s^i]\left(s^m + \sum_{i=0}^{m-1} c_{n+1} s^i \right) x(s) =$$

$$= -bk \sum_{i=0}^{n-1} c_i s^i x(s) + bkv(s)\,.$$

Now we open up the parentheses in the left side and group terms by powers of s. The terms having no bk will combine (denote them as $\sigma(\cdot)$):

$$[(1+bk)s^{n+m} + ((1+bk)c_{n+m-1} + \sigma(\cdot))s^{n+m-1} + ((1+bk)c_{n+m-2} + \sigma(\cdot))s^{n+m-2} + \ldots$$

$$\ldots((1+bk)c_n + \sigma(\cdot))s^n + \sigma(\cdot)s^{n-1} + \ldots + \sigma(\cdot)s + \sigma(\cdot).]x(s) = (-bk\sum_{i=0}^{n-1} c_i s^i)x(s) + bkv(s)\,.$$

From here we get:

$$[(1+bk)s^{n+m} + \sum_{i=0}^{m-1}((1+bk)c_{n+i} + \sigma(\cdot))s^{n+i} + \sum_{i=0}^{n-1}(bkc_i + \sigma(\cdot))s^i]x(s) = bkv(s)\,.$$

With the increase of k in the left side we get polynomial (21). Lemma is proved.

Exapmple 2.1.

For the object describes by the equation

$$x^{(2)} + a_1 x^{(1)} + a_0 x = bu\,,$$

it is required to design such regulator that the characteristic polynomial of the system would have the form

$$N_3(s) = s^3 + c_2 s^2 + c_1 s + c_0\,.$$

Let us make use of Lemma 1. Here $n = 2$, $n + m = 3$. We construct the regulator of the form

$$z^{(1)} + c_2 z + c_1 x^{(1)} + c_0 x = v\,,$$

$$u = k(z - x^{(2)})\,.$$

For the realization of the obtained control law the vector $x, x^{(1)}, x^{(2)}$ is necessary. If we take $m = 0$ in (20), we get corollary 1:

Corollary 1. For the object (18) with regulator (20):

$$u(s) = (-N(s)x(s) + v(s))\,, \quad (20^*)$$

where $N(s) = \sum_{i=0}^{n} c_i s^i$, $c_n = 1$, characteristic polynomial of the system (18) and (20*) approaches asymptotically with the growth of k to

$$\sum_{i=0}^{n} c_i s^i\,,\ c_n = 1. \quad (21^*)$$

Example 2.2.

For the object $sx(s) = u(s)$ it is necessary to design the regulator such that $n(s) = s + c_0$. According to (20*) we take

$$u(s) = k(-(s + c_0)x(s) + c_0 v(s))\,.$$

The factor c_0 before $u(s)$ will provide the transfer factor equal to 1. Then

$$sx(s) = k(-(s + c_0)x(s) + c_0 v(s))\,.$$

or

$$((\mu + 1)s + c_0)x(s) = c_0 v(s)\,,\ \mu = k^{-1}\,.$$

With small μ we have

$$(s + c_0)x(s) = c_0 v(s)\,.$$

If we consider this problem via disturbance channel $f(t)$ then

$$sx(s) = u(s)\,,$$

$$\mu u(s) = -(s + c_0)y(s) + c_0 v(s)\,,$$

$$y(s) = f(s) + x(s)\,,$$

we get

$$(s + c_0)y(s) = \mu s f(s)\,.$$

As a rule, in practice, not the full state vector is accessible for measurement but only its part. Let us consider the case where the only signal x(t) is available. The regulator (20) can be presented in the form

$$u(s) = k(v(s) - \sum_{i=0}^{n+m-1} c_i s^i x(s))(\sum_{j=0}^{m} c_{n-j} s^{j-1})$$

Here, unlike the expression (20) the only signal x(s) is used, and its derivatives are obtained with the help of differentiating regulator. In order to have this regulator physically realizable, the power of its numerator should not exceed the denominator power. For this purpose we construct the regulator in the form

$$d(s)u(s) = k(-\sum_{i=0}^{m} c_i s^i x(s) + v(s))\,, \quad (22)$$

$$d(s) = \sum_{i=2}^{n} \alpha_i \mu^{i-1} s^{m-n+1} + \sum_{i=0}^{m-n+1} \alpha_i s^i\,. \quad (23)$$

where $m > n$, $\mu = (bk)^{-1}$, $\alpha_n = 1$, $\alpha_{m-n+1} = 1$.

Lemma 2.

Characteristic equation of the system (18), (22), (23) with the growth of k is arbitrary exactly described by the expression

$$\sum_{i=1}^{n} \alpha_i \mu^i s^{m+i} + \sum_{i=0}^{m} c_i s^i = 0, \quad (24)$$

where $\alpha_n = 1$.

Proof.

The substitution of (22), (23) into (18) gives

$$(s_n + a_{n-1}s^{n-1} + ... + a_1 s + a_0)(\mu^{n-1}s^m + \alpha_{n-1}\mu^{n-2}s^{m-1} + ... + s^{m-n+2}\alpha_2\mu + s^{m-n+1} + \alpha_{m-n}s^{m-n} + ...$$
$$... + \alpha_1 s + \alpha_0)x(s) = -bk(c_m s^m + ... + c_1 s + c_0)x(s) + bkv(s). \quad (25)$$

With an increase of k the terms containing multiplier μ, turn to arbitrary small values compared to similar terms without μ. The terms contain *s* in power *m* + 1 and higher have no similar terms without multiplier; therefore they produce essential effect on the roots of characteristic equation (25). Thus, at $v(s) = 0$, $k \to \infty$ the roots of equation (25) are arbitrary exactly approaching the roots of equation (24).

As it is seen, factors α_i for $i < m$ do not make influence on the system characteristic equation. There are chosen from condition of physical realization possibility for regulator (22), (23). The factors are included into characteristic equation (24), therefore they should be selected on the base of the system root stability. With an increase in *k* equation (24) is reduced to the desired equation. Additional *n* roots exceed by modulus the desired roots. These roots correspond to fast motion of the system. In order to neglect their influence one has to provide faster damping of modes originated by these roots compared to the damping of desired modes. One can use Euler parameter [14]. The sufficient condition for the left half-flat in sector ±60° near the real axis has the form [15]:

$$\delta_i = k_i^2 (k_{i-1}k_{i+1})^{-1} \geq 2,$$
$$i = 2,...,n+m-1.$$

That condition is provided, if

$$c_i^2 (c_{i-1}c_{i+1})^{-1} \geq 2, \text{ for } i = 1,...,m-1. \quad (26)$$

$$\alpha_j^2 (\alpha_{j-1}\alpha_{j+1})^{-1} \geq 2, \text{ for } j = 2,...,n-1. \quad (27)$$

$$(\alpha_2 c_m)^{-1} \geq 2, \; c_m^2 (\mu c_{m-1})^{-1} \geq 2. \quad (28)$$

Taking into account $\alpha_0 = 1$, $\mu << 1$, we get independent and similar in their form systems of limits for α_j and c_i. For the sake of simplicity one can take the full equality in (26) and (27) and search for the solution in the form

$$\alpha_j = 2^{l(j)}, \; j = 1,...,n-1.$$

Then we get the system of $n+1$ equations:

$$2l(i) - l(i-1) - l(i+1) = 1, \; j = 1,...,n-1,$$
$$l(0) = 1, \; l(n) = 1,$$

for $n+1$ unknown values which is easily solvable by the method of substitutions. Factors c_i are calculated similarly.

Let us consider the general case where in the object (18) only value *x* and its *r* derivatives ($r \leq n$) are accessible for measurement. Let us construct the regulator in the form:

$$u(s) = k(v(s) - \sum_{i=0}^{r} c_i x^{(i)}(s) -$$
$$- \sum_{i=1}^{m+n-r} c_{i+r} s^i x^{(r)}(s))d^{-1}(s), \quad (29)$$

$$d(s) = \sum_{j=1}^{n-r-1} \alpha_{j+1}\mu^j s^{m+j+1} + \sum_{i=0}^{m+1} \beta_i s^i, \quad (30)$$

where on coefficients α_j the restrictions (27) for $j = 1,2,3...,n-r-2$ are imposed and c_i satisfy the limitation (26) for $i = 1,2,3,...,m+n-1$.

Theorem 1.

The dynamics of system (18) with regulator (29), (30) with the growth of *k* differs arbitrarily small from that of the system described by the equation

$$(\sum_{j=1}^{n-r} \alpha_j \mu^j s^{m+j+1} + N(s))x(s) = v(s). \quad (31)$$

Proof.

Let us substitute (29), (30) in (18).

$$\sum_{i=0}^{n} a_i s^i x(s) = bk(v(s) - \sum_{i=0}^{r} c_i x^{(i)}(s) - \sum_{i=1}^{m+n-r} c_{i+r} s^i x^{(r)}(s))d^{-1}(s),$$
$$\mu \sum_{j=0}^{n} a_j s^j x(s)(\sum_{i=1}^{n-r-1} \alpha_{i+1}\mu^i s^{m+i+1} + \sum_{i=0}^{m+1} \beta_i s^i) = v(s) - \sum_{i=0}^{m+n} c_i s^i x(s). \quad (32)$$

The growth of k leads to a decrease in $\mu = b^{-1}k^{-1}$ down to any arbitrary small value. In this case, $m+n$ roots of equation (32) are arbitrarily exactly approach to roots of equation (31) and all the rest of $n+r$ roots move along the negative direction within the limits of sector ±60° near the real axis. The system reaction to any action is defined as a sum of reaction of single modes corresponding to the roots of equation (32). With an increase of k, $m+n$ modes coincide arbitrarily exactly with those of the system (31) and the contribution of all the rest of modes into transient processes can be arbitrarily decreased. Consequently the dynamics of system (18), (29), (30) is arbitrarily exactly approaching the dynamics of the system (31).

Example 2.3.

For the object

$$a_2 x^{(2)} + a_1 x^{(1)} + x = u$$

it is necessary to design such regulator that the characteristic equation of the system (slow motion) would have the form

$$Cs^3 + Bs^2 + As + 1 = 0\,.$$

According to Lemma 2 at $n=2$, $m=3$ one gets the regulator

$$u(s) = k(-(Cs^3 + Bs^2 + As + 1)d^{-1}(s)x(s) +$$
$$+ d^{-1}(s)v(s)\,.$$

Here

$$d(s) = k^{-1}s^3 + s^2 + \sqrt{2}s + 1\,.$$

Values α_i are chosen in accordance with (27) from the condition of location of regulator roots in sector ±60° in the left half-plane. The characteristic polynomial of system is equal to the value:

$$\mu^2 s^5 + \mu s^4 + Cs^3 + Bs^2 + As + 1\,.$$

Corollary 2.

The characteristic polynomial of system with object (18) and regulator

$$d(s)u(s) = k(-c(s)x(s) + c_0 v(s))\,,$$

$$c(s) = \sum_{i=0}^{n} c_i s^i\,, \quad (12^*)$$

$$d(s) = \sum_{i=2}^{n} \alpha_i \mu^{i-1} s^i + s + \alpha_0\,,\ \alpha_n = 1,$$

$$\alpha_n = 1,\ c_n = 1,\ (13^*)$$

asymptotically approaches to the form:

$$\sum_{i=2}^{n} \alpha_i \mu^i s^{n+i} + \sum_{i=0}^{n} c_i s^i\,,\ \alpha_n = 1,\ \alpha_n = 1,$$

$$c_n = 1\,.\ (14^*)$$

This corresponds to the case when the order of the desired characteristic polynomial of system coincides with the object order. For the proof of corollary 2 it is sufficient to substitute $m=n$ in (22), (23) and (24). For $n=1$ the first term in (22) and (23) vanishes.

Example 2.4.

Let it be necessary for the object

$$(s^2 + s + a_0)x(s) = bu(s)$$

to get the characteristic polynomial of the system equal to

$$s^2 + c_1 s + c_0\,.$$

According to Corollary 2 we take the regulator in the form

$$(\mu s^2 + s + \alpha_0)u(s) =$$
$$= k(-(s^2 + c_1 + c_0)x(s) + c_0 v(s))\,.$$

Then the system characteristic polynomial is the following:

$$\mu^2 s^4 + \mu s^3 + s^2 + c_1 s + c_0$$

and with μ sufficiently small one can regard that the problem is solved.

Example 2.5.

For the first order object

$$(s + a_0)x(s) = b_0 u(s)$$

let us make of Corollary 2 for selecting the regulator:

$$\mu(s + \alpha_0)u(s) = b_0^{-1}(-(s + c_0)x(s) + c_0 v(s))\,.$$

Then with μ sufficiently small the system is described by the equation

$$(\mu s^2 + s + c_0)x(s) = c_0 v(s)\,.$$

That is the slow motions are described by equation of the first order

If it is necessary to design astatic system, one should use artificial technique, namely, first introduce an integrator into object and then to design regulator for the new object of higher order.

Example 2.6.

Let us consider object from Example 2.5. Let us introduce an integrator into the object and get:

$$(s^2 + a_1 s)x(s) = b_0 u(s)\,.$$

Let us consider the regulator for this new object:

$$\mu u(s) = b_0^{-1}(-(s + c_0)x(s) + c_0 v(s))\,.$$

Further we find out the characteristic polynomial of the system

$$\mu(s^2 + a_1)x(s) = -(s + c_o)x(s) + c_0 v(s)\,.$$

Hence

$$(\mu s^2 + s + c_o)x(s) = c_0 v(s)\,.$$

So the real regulator has the form

$$\mu s u(s) = b_0^{-1}(-(s + c_0)x(s) + c_0 v(s))\,.$$

3. SYSTEM WITH SINGLE INPUT AND OUTPUT (SISO): DECREASE OF THE ORDER

Let it be necessary to decrease the order of the system (18), i. e. $m < n$, where m is the object order. In this case, in the characteristic equation first $m+1$ coefficients could be given equal to those of desired equation and all the rest $n-m$ should be proportional to k^{-1} in power 1 and higher. In order to have the root of the system with regulator within the sector ± 60°, we write down the characteristic equation in the form

$$\sum_{i=1}^{n-m} \alpha_i k^{-i} s^{m+i} + N_m(s) = 0,$$

$$N_m(s) = \sum_{i=0}^{m} c_i s^i . \quad (33)$$

For the object with fully measurable state vector let us construct the regulator

$$u(s) = k^{n-m} b^{-1} \left(-\sum_{i=1}^{n-m-1} \alpha_i k^{-i} s^{m+i} x(s) + v(s)\right) . \quad (34)$$

Here $n \geq m+1$, $\alpha_1 = 1$, $c_0 = 1$.

Lemma 3.

With the growth of *k* the characteristic equation of the system (18), (34) coincides arbitrarily exactly with equation (33).

Proof.

The substitution of (34) into (18) with $k^{-1} = \mu$ yields

$$\mu^{n-m}\left(s^n + \sum_{i=0}^{n-1} \alpha_i s^i\right) x(s) =$$

$$= -\sum_{i=1}^{n-m-1} \alpha_i \mu^i s^{m+i} x(s) - \sum_{i=0}^{m} c_i s^i x(s) + v(s) .$$

From the polynomial before *x*(*s*) is equal to

$$\mu^{n-m} s^n + \mu^{n-m} a_{n-1} s^{n-1} + \ldots + \mu^{n-m} a_0 + \alpha_{n-m-1} \mu^{n-m-1} s^{n-1} + \alpha_{n-m-2} \mu^{n-m-2} s^{n-2} + \ldots + \alpha_1 \mu s^{m+1} + c_m s^m + \ldots$$
$$\ldots + c_m s^m + \ldots + c_1 s + c_0 .$$

That coincides with (33) at $\mu \to 0$. Lemma is proved.

Corollary 3.

For $n-m=1$ and regulator of the form

$$u(s) = k\left(v(s) - \sum_{i=0}^{n-1} c_i s^i x(s)\right) b^{-1}$$

the system equation has the form

$$\left(\mu s^n + \sum_{i=0}^{n-1} c_i s^i\right) x(s) = v(s) .$$

The slow motion of the system are determined by the second term of this equation and the condition of the first term can arbitrarily be deceased due to μ.

Example 3.1.

For the object

$$(s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0) x(s) = bu(s)$$

one must construct the regulator such that the characteristic polynomial has the form

$$N_2(s) = c_2 s^2 + c_1 s + 1 .$$

According to Lemma 3, the regulator should have the form

$$u(s) = \mu^{-2} b^{-1} (v(s) - (\mu s^3 + c_2 s^2 + c_1 s + 1) x(s)) .$$

Then the characteristic polynomial of the system

$$N(s) = \mu^2 s^4 + \mu s^3 + c_2 s^2 + c_1 s + 1 .$$

And as $\mu \to 0$ with the growth of *k*, it approaches the form

$$N(s) = c_2 s^2 + c_1 s + 1 .$$

Example 3.2.

For the object

$$(s^2 + a_1 s + a_0) x(s) = bu(s)$$

It is necessary to find out regulator that slow motions would be described by the first order equation. According to Corollary 3, let us take the regulator

$$u(s) = k(v(s) - (c_1 s + 1) x(s)) b^{-1} .$$

The system equation

$$(\mu s^2 + (\mu a_1 + c_1) s + (\mu a_0 + 1)) x(s) = v(s) .$$

With a small μ it is close to the following equation

$$(\mu s^2 + c_1 s + 1) x(s) = v(s) .$$

The problem is solved.

Let us consider the case, where only the signal *x* and its *r* derivatives ($n > n > r > 0$) are accessible for measurements. Let us construct the regulator:

$$\sum_{i=0}^{n-r-1} \mu^i \alpha_{n-m+i} s^i u(s) = k^{n-m} b^{-1} \left(-\sum_{i=1}^{n-m-1} \mu^i \alpha_i s^{m-r+i} x^{(r)}(s) - \sum_{i=1}^{m-r} c_{r+i} s^i x^{(r)}(s) - \sum_{i=0}^{r} c_i s^i x(s) + v(s)\right), \quad (35)$$

$\alpha_1 = 1$. In this case, in the given expression we shall have the following conventional representation:

$$s^i x(s) = \begin{cases} x^{(i)}(s), & \forall i \leq r, \\ s^{i-r} x^{(r)}(s), & \forall i > r. \end{cases}$$

This means that the regulator uses all the available signals and deficient derivatives are obtained with the help of derivative amplifiers. The introducing into the regulator of denominator of the order $n-r$ enables one to assume that the regulator is physically realizable.

Theorem 2.

For the object (18) and regulator (35) the system equation with the growth of k is arbitrarily exactly described by the expression

$$\sum_{i=0}^{2n-m-r-1}\alpha_i k^{-i}s^{m+i}x(s)+\sum_{i=1}^{m}c_i s^i x(s)=v(s).$$

Proof.

Substituting (35) in (18) one can get

$$k^{m-n}\sum_{i=0}^{n}\alpha_i s^i x(s)\sum_{j=0}^{n-r-1}\alpha_{n-m+j}k^{-j}s^j =$$

$$=-\sum_{i=1}^{n-m-1}\alpha_i k^{-i}s^{m+i}x(s)-\sum_{i=0}^{m}c_i s^i x(s)+v(s)$$

.

The left side terms containing s in power $i+j$ are not more $n-1$ with the growth of k are negligibly small compared to similar terms in the right side. The terms of higher order combine by power of s and separate those containing multipliers k^{-1} in the lowest power. The contribution of all the rest of terms with the growth of the coefficient k decreases arbitrarily. With $k\to\infty$ the system equation has the form (32).

Let us consider a special case of Theorem 2 where only the output signal $x(s)$ is accessible for measurement. Used below n is an order of the object and m is the order of slow motions equation.

Lemma 4.

For the regulator

$$u(s)=\mu^{m-n}b^{-1}d^{-1}(s)(-\sum_{i=1}^{n-m-1}\alpha_i\mu^i s^{m+i}x(s)-$$

$$-\sum_{i=0}^{m}c_i s^i x(s)+v(s), \quad (37)$$

where

$$d(s)=\sum_{i=0}^{n-1}\alpha_{n-m+i}\mu^i s^i,$$

equation of the system (18) and (37) with the growth of k approaches the form

$$(\sum_{i=1}^{2n-m-1}\alpha_i\mu^i s^{m+i}+\sum_{i=0}^{m}c_i s^i)x(s)=v(s). \quad (38)$$

Proof.

The proof is similar to those of preceding statements. Let us give it here just to show how to formalize such equations.

The system equation is

$$\mu^{n-m}\sum_{i=0}^{n-1}\alpha_{n-m+i}\mu^i s^i\sum_{i=0}^{n}a_i s^i=-\sum_{1}^{n-m-1}\alpha_i\mu^i s^{i+m}x(s)-\sum_{i=0}^{m}c_i s^i x(s)+c_0 v(s).$$

All the complexity is in the left side. Let exposure

$$\alpha_{2n-m-1}\mu^{n-1}a_n s^{2n-1}+\alpha_{2n-m-2}\mu^{n-2}s^{2n-2}+\ldots+$$
$$+\alpha_{n-m}a_n s^n+\alpha_{2n-m-1}\mu^{n-1}s^{2n-2}+\ldots+$$
$$+\alpha_{n-m-1}\mu a_{n-1}s^n+\alpha_{n-m}a_{n-1}s^{n-1}+$$
$$+\alpha_{2n-m+1}\mu^{n-1}a_1 s^n+\alpha_{2n-m-2}\mu^{n-2}a_1 s^{n-1}+\ldots$$

The conventionally obtained table is shown in Fig.12. Consider the terms by columns. Taking into account that $\mu^i>>\mu^{i+1}$, one can get the lemma statement.

Corollary 3.

For the regulator

$$u(s)=(\mu bd(s))^{-1}(-\sum_{i=0}^{n-1}c_{i-1}s^i x(s)+v(s)). \quad (37^*)$$

Here

$$d(s)=\sum_{i=0}^{n-1}\alpha_i\mu^i s^i.$$

Characteristic polynomial of the system (18), (37*) is equal to

$$\sum_{i=1}^{n}\alpha_i\mu^i s^{n+i-1}+\sum_{i=0}^{n-1}a_i s^i.$$

Example 3.3.

For the object

$$(s^3+a_2s^2+a_1s+a_0)x(s)=u(s)$$

the regulator is necessary which provides the characteristic polynomial of system equal to $N_1(s)=c_1(s)+1$.

Here $n=3$, $m=1$, the regulator has the form:

$$u(s)=\mu^{-2}b^{-1}\left(-\frac{\mu s^2+c_1s+1}{\mu^2s^2+2\mu s+1}x(s)+\frac{1}{\mu^2s^2+2\mu s+1}v(s)\right).$$

The characteristic polynomial of the system with an account of fast motions is

$$N(s)=\mu^4s^5+2\mu^3s^4+2\mu^2s^3+\mu s^2+c_1s+1$$

.

The slow motions are described by the required characteristic polynomial.

Example 3.4.

For the object

$$(s^2 + a_1 s + a_0)x(s) = u(s)$$

and regulator

$$u(s) = \mu^{-1} b^{-1}\left(-\frac{As+1}{\mu s+1} x(s) + \frac{1}{\mu s+1} v(s) \right)$$

the system equation will be

$$(\mu^2 s^3 + \mu s^2 + As + 1)x(s) = v(s).$$

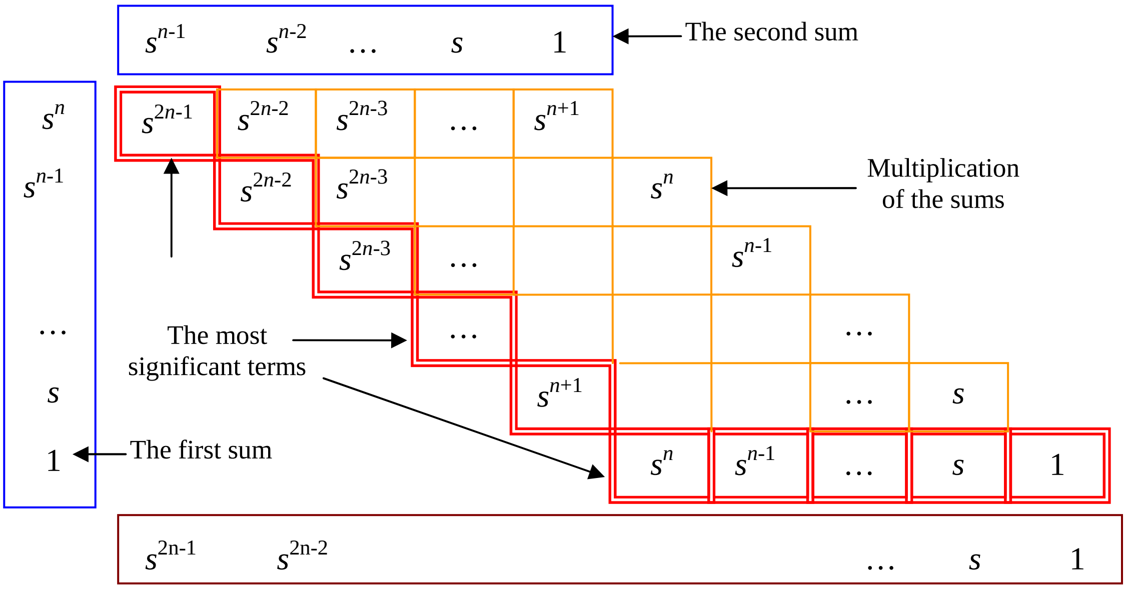

Fig. 12. Schematic of calculations corresponding to multiplies polynomials

CONCLUSION

In the paper the development of method of localization formulated in the works by A.S. Vostrikov is proposed. The set of regulators is suggested which enable controlling of objects whose parameters are known insufficiently exactly of slowly vary with time. Also the paper gives possibility to approach the desired dynamic properties of the system with the demanded accuracy. The obtained order of the system in the terminology of slow motions can be higher of lower than the object order. In practice the necessary value of the gain is not too large. Since in the majority of cases the equation coefficients for object (1) are varied no more than the order of magnitude, for obtaining good approximation (of 5 %) to linear model, the value of gain $k = 20...100$ is enough.

The given formulae for calculation of regulators for various cases are illustrated by many examples. For the calculation one only needs to define the requirements to the device, evaluate the order of equation describing the object and its parameters and then to choose and approximate form of regulator, For this purpose one should use one of the lemmas, theorems or to make use of one of corollaries. After that the only thing is left to select the value of tuning parameters either directly on the device model or by the modelling method.

In conclusion the authors would like to underline that the given relations could be easily extended to the case of using microprocessors in control device. For that one has to go from the differential equations to difference equations.

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